Trigonometric Functions

Assertion Reason Questions

Direction: In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R).

Choose the correct answer out of the following choices.

(a) Both (A) and (R) are true and (R) is the correct explanation of (A).

(b) Both (A) and (R) are true but (R) is not the correct explanation of (A).

(c) (A) is true but (R) is false.

(d) (A) is false but (R) is true.

1. Assertion (A): The radius of the circle in which a central angle of 60° intercepts an arc of length 44 cm is 42 cm.

Reason (R): Length of an arc of a circle is

L = rθ, where r is non-italic

angle.

Ans. (a) Both (A) and (R) are true and (R) is the correct explanation of (A).

Explanation: Here, $l = 44$ cm and $\theta = 60° = \frac{\pi}{3}$

As we know that

$$l = r\theta$$

$$\therefore \quad r = \frac{l}{\theta}$$

$$= \frac{44 \times 3}{\pi}$$

$$= \frac{44 \times 3 \times 7}{22} = 42 \text{ cm}$$

2. Assertion (A): Value of sin (-270)° is 1.

Reason (R): sin (180° + θ) = sin θ.

Ans. (a) Both (A) and (R) are true and R is the correct explanation of (A).

Explanation: sin(-270°)=sin (180° +90°)

We know that

sin (180°+θ)=sin θ

=(-sin 90°) = 1

3.

Assertion (A): The value of $\theta = \frac{\pi}{3}$ or $\frac{2\pi}{3}$, when θ lies between (0, 2π) and $\sin^2 \theta = \frac{3}{4}$.

Reason (R): sin θ is positive in the first and second quadrant.

Ans. (d) (A) is false but (R) is true.

Explanation: Given, $\sin^2 \theta = \frac{3}{4}$

$\Rightarrow \sin \theta = \frac{\sqrt{3}}{2}$ or $-\frac{\sqrt{3}}{2}$.

Case I: When $\sin \theta = \frac{\sqrt{3}}{2} = \sin \frac{\pi}{3}$

$\Rightarrow \sin \theta = \sin \frac{\pi}{3}$ or $\sin (\pi - \frac{\pi}{3})$

$\Rightarrow \theta = \frac{\pi}{3}$ or $\pi - \frac{\pi}{3}$,

i.e., $\theta = \frac{\pi}{3}$ or $\frac{2\pi}{3}$

Case II: When $\sin \theta = -\frac{\sqrt{3}}{2}$, then θ lies either in the third or fourth quadrant.

Now, $\sin \theta = -\frac{\sqrt{3}}{2} = -\sin \frac{\pi}{3}$

$= \sin \left(\pi + \frac{\pi}{3}\right)$ or $\sin \left(2\pi - \frac{\pi}{3}\right)$

$\theta = \pi + \frac{\pi}{3}$ or $2\pi - \frac{\pi}{3}$

$\Rightarrow \theta = \frac{4\pi}{3}$ or $\frac{5\pi}{3}$,

Hence, $\sin^2 \theta = \frac{3}{4}, 0 < \theta < 2\pi$

$\Rightarrow \theta = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$,

4. Let sec θ + tan θ=m, where 0 <m<1.

Assertion (A): $\sec\theta = \dfrac{m^2+1}{2m}$ and

$$\sin\theta = \frac{m^2-1}{m^2+1}$$

Reason (R): > lies in the third quadrant.

Ans. (c) (A) is true but (R) is false.

Explanation: Given, sec θ + tan θ = m,

where, 0 < m < 1 ...(i)

We know that, $\sec^2\theta - \tan^2\theta = 1$ ...(ii)

dividing (ii) by (i), we get

Also, $\sin\theta = \tan\theta\cos\theta = \dfrac{\tan\theta}{\sec\theta} = \dfrac{m^2-1/2m}{m^2+1/2m}$

$$= \frac{m^2-1}{m^2+1}$$

5. Let a be a real number lying between 0 and

$\frac{\pi}{2}$ **and *n* be a positive integer.**

Assertion (A): $\tan\alpha + 2\tan 2\alpha + 2^2\tan 2^2\alpha + \ldots + 2^{n-1}\tan 2^{n-1}\alpha + 2^n\cot 2^n\alpha = \cot\alpha$

Reason (R): $\cot\alpha - \tan\alpha = 2\cot 2\alpha$.

Ans. (a) Both (A) and (R) are true and (R) is the correct explanation of (A).

Explanation:

Given, $\cot\alpha - \tan\alpha = \dfrac{1}{\tan\alpha} - \tan\alpha = \dfrac{1-\tan^2\alpha}{\tan\alpha}$

$$= 2\left(\frac{1-\tan^2\alpha}{2\tan\alpha}\right) = 2\cot 2\alpha$$

From here, we get $\tan\alpha = \cot\alpha - 2\cot 2\alpha$

Making repeated use of this identify, we shall obtain

$\tan\alpha + 2\tan 2\alpha + 2^2\tan 2^2\alpha + \ldots + 2^{n-1}\tan 2^{n-1}\alpha + 2^n\cot 2^n\alpha$

$= (\cot\alpha - 2\cot 2\alpha) + 2(\cot 2\alpha - 2\cot 2^2\alpha) + 2^2(\cot 2^2\alpha - 2\cot 2^3\alpha) + \ldots + 2^{n-1}(\cot 2^{n-1}\alpha - 2\cot 2^n\alpha) + 2^n\cot 2^n\alpha = \cot\alpha$

6. Assertion (A): The value of sin (-690°) cos (-300°) + cos (-750°) sin (-240°) = 1

Reason (R): The value of sin and cos is negative in the third and fourth quadrant respectively.

Ans. (c) (A) is true but (R) is false.

Explanation: sin (-690°)=-sin 690°

=-sin (2 x 360° - 30°)

7. If A + B + C = 180°, then

Assertion(A): $\cos^2\frac{A}{2}+\cos^2\frac{B}{2}-\cos^2\frac{C}{2}$

$$= 2\cos\frac{A}{2}\cos\frac{B}{2}\sin\frac{C}{2}$$

Reason (R): $\cos C + \cos D$

$$= 2\cos\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$$

Ans. (a) Both (A) and (R) are true and (R) is the correct explanation of (A).

Explanation: Given, $\cos^2\frac{A}{2}+\cos^2\frac{B}{2}-\cos^2\frac{C}{2}$

$$= \frac{1+\cos A}{2}+\frac{1+\cos B}{2}-\frac{1+\cos C}{2}$$

$$= \frac{1+(\cos A+\cos B-\cos C)}{2} \quad \text{...(i)}$$

Now, $\cos A + \cos B - \cos C$

$$= 2\cos\frac{A+B}{2}\cos\left(\frac{A-B}{2}\right)-\cos\left(2\cdot\frac{C}{2}\right)$$

$$= 2\sin\frac{C}{2}\cos\left(\frac{A-B}{2}\right)-\left(1-2\sin^2\frac{C}{2}\right)$$

$$\left[\because \cos\left(\frac{A+B}{2}\right)=\cos\left(90°-\frac{C}{2}\right)=\sin\left(\frac{C}{2}\right)\right]$$

$$= 2\sin\frac{C}{2}\left\{\cos\left(\frac{A-B}{2}\right)+\sin\left(\frac{C}{2}\right)\right\}-1$$

$$= -1+2\sin\frac{C}{2}\left\{\cos\left(\frac{A-B}{2}\right)+\cos\left(\frac{A+B}{2}\right)\right\}$$

$$= -1+4\sin\left(\frac{C}{2}\right)\cos\left(\frac{A}{2}\right)\cos\left(\frac{B}{2}\right) \qquad ...(ii)$$

From (i) and (ii), we get

L.H.S of the given identity

$$= \frac{1+\left(-1+4\cos\left(\frac{A}{2}\right)\cos\left(\frac{B}{2}\right)\sin\left(\frac{C}{2}\right)\right)}{2}$$

$$= 2\cos\left(\frac{A}{2}\right)\cos\left(\frac{B}{2}\right)\sin\left(\frac{C}{2}\right)$$